The odderon in high energy elastic pp scattering[*]

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Abstract. We study the odderon contribution to elastic pp and $p\bar{p}$ scattering at high energies. Different models for the odderon–proton coupling are considered and their effects on the differential cross section in the dip region are investigated. We use a Regge fit by Donnachie and Landshoff as a framework and replace its odderon contribution by the different models. We consider two models for the odderon–proton impact factor proposed by Fukugita and Kwieciński and by Levin and Ryskin. In addition we construct a geometric model of the proton which allows us to put limits on the size of a possible diquark cluster in the proton. All models are able to describe the data well. The two models for the impact factor require the strong coupling constant to be fixed rather precisely. In the geometric model a relatively small diquark size is required to describe the data.

1 Introduction

The odderon is an interesting but elusive object. Its history goes back to 1973 when the possible contribution of an exchange carrying negative C parity to very high energy collisions was first discussed [1]. The leading contribution to hadronic scattering processes is in general well described by the exchange of a pomeron with intercept $\alpha \simeq 1.09$, resulting in slowly increasing cross sections, $\sigma \sim s^{\alpha-1}$ [2]. The pomeron carries vacuum quantum numbers and therefore leads to a high energy behaviour of hadronic cross sections that is equal for pp and $p\bar{p}$ scattering. In lowest order in QCD the pomeron can be identified with the exchange of two gluons in a colour singlet state. The odderon is the $C = -1$ partner of the pomeron. In lowest order it can be understood as the exchange of three gluons in a symmetric colour singlet state. As in the case of the pomeron, the odderon exchange gives a contribution to the cross section that behaves like a power of the energy, $s^{\alpha_{\mathcal{O}}-1}$. The odderon intercept $\alpha_{\mathcal{O}}$ is expected to be close to one — in contrast to the intercept of $C = -1$ reggeon exchanges, which is around 0.5. For a review of the historic roots of the odderon and some relevant references we refer the reader to [3].

The experimental evidence for the existence of an odderon, however, remains rather scarce. For a long time the odderon search concentrated on observing a difference between the cross sections for pp and $p\bar{p}$ scattering at high energies. The odderon causes such a difference because it carries negative C parity and thus gives opposite contributions to these cross sections. For an intercept larger than 1 this difference should in fact increase with energy and give a visible effect. But recent perturbative results indicate that the intercept is smaller than or equal to 1, so that the difference does not increase. The experimental data actually disfavour a sizable effect of the odderon in this difference. More sensitive to the odderon exchange than the cross section is the ratio of the real to the imaginary part of the scattering amplitude in the forward direction. But also in this case no indication of an odderon exchange has been found [4]. To date the only evidence for the existence of the odderon is found in the t-dependence of pp and $p\bar{p}$ elastic cross sections at high energy. The pp data show a characteristic dip at $|t| \simeq 1.3\,\mathrm{GeV}^2$, whereas the $p\bar{p}$ data only flatten off at that t. Unfortunately, there are only few $p\bar{p}$ data available [5,6]. Figure 1 shows the data [5] in the relevant region of the dip. A good description of all available data [5–9] for elastic pp scattering was given by Donnachie and Landshoff [10]. In this description (see also Sect. 2.1 below) the presence or absence of the dip originates from the exchange of the odderon. Also the behaviour of the elastic cross sections at large t is well described by odderon exchange. A number of aspects of elastic pp and $p\bar{p}$ scattering has been discussed in the literature particularly in the light of odderon physics; see for example [11–18]. No other successful description of the data without an odderon has been found so far.

Recently there has been renewed interest in the odderon which has especially concentrated on two areas. One of these areas is the perturbative treatment of the odderon

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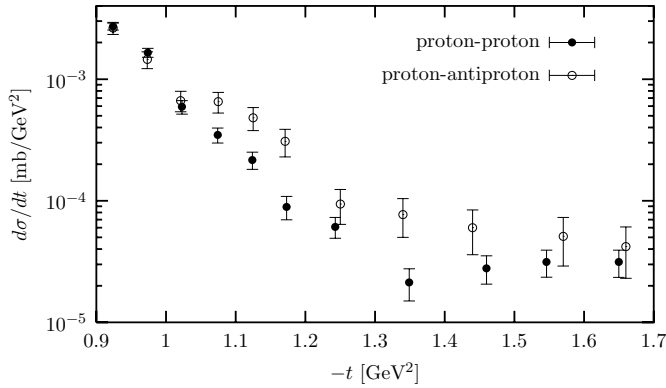**Fig. 1.** Differential cross section for elastic pp and $p\bar{p}$ scattering in the dip region for $s^{1/2} = 53\,\mathrm{GeV}$; data taken from [5]

and in particular the determination of the odderon intercept [19–27]. Perturbation theory can be applied to this problem if a scattering process involves a large momentum scale. The perturbative odderon is described by the Bartels–Kwieciński–Praszałowicz equation [28,29] which resums large logarithms of the centre-of-mass energy. It has the form of a Bethe–Salpeter type equation for the interaction of a system of three reggeized gluons in the t-channel. The interaction of the gluons induces a non-trivial energy dependence of the cross section, i.e. it leads to an intercept different from one. A number of interesting aspects of the perturbative odderon has been studied in [30–43]. In [31,32] it was shown that this system of three reggeized gluons is equivalent to an integrable model. Eventually the study of this system lead to the determination of its ground state energy [25,26]. It was found to correspond to an intercept slightly below 1. There exists, however, a special solution with intercept exactly equal to 1 [27]. In [44] even the whole energy spectrum of the perturbative odderon was found. Another aspect of the perturbative odderon is its rôle played in the theory of unitarity corrections to the perturbative (BFKL) pomeron [45,46]. This problem was addressed in [47] where the perturbative pomeron–odderon–odderon vertex was calculated.

The other area on which interest has concentrated is to find more exclusive processes in which the odderon contribution should be dominant. Some processes have been considered which can be calculated perturbatively. The most interesting among them is the diffractive photo- or electroproduction of η_c or other heavy pseudoscalar mesons at HERA [48–50]. If these mesons have charge parity +1 pomeron exchange cannot contribute to their photo- or electroproduction [51]. However, the corresponding cross sections are estimated to be rather small, in the range of several tens of pb or even lower. Much larger cross sections are expected for the diffractive production of light pseudoscalar and tensor mesons [52,53]. In this case the theoretical predictions require the use of nonperturbative methods and models [54–56]. But recent measurements of pion photoproduction by the H1 collaboration did not show any signs of the odderon [57,58]. The reasons for this failure of the prediction are not clear at the moment, and

the presence of this process should in fact not be specific to the model assumptions.

Recently the investigation was proposed of certain charge asymmetries in diffractive processes [59,60]. These asymmetries arise from pomeron–odderon interference and are expected to provide a good chance of finding the odderon at HERA. Another process of interest will be the quasidiffractive production of η_c (or other pseudoscalar or tensor) mesons in collisions of real or virtual photons at a future linear collider like TESLA [61]. An interesting process for the odderon search is also double-diffractive production of vector mesons at Tevatron or at the LHC [62].

The apparent absence of the odderon in the photoproduction of pions mentioned above is rather surprising. Its cause is an open question which clearly needs to be clarified. One obvious possibility is that for some reason the nonperturbative model used here does not work properly in this particular situation and that the cross section has thus been overestimated. This uncertainty can be excluded in perturbative situations. Therefore the investigation of perturbative processes involving the odderon becomes even more important.

A theoretical uncertainty that is inherent even in the perturbatively calculable processes like diffractive η_c production is the exact form of the coupling of the odderon to the proton, i.e. the odderon–proton impact factor. A few very general facts about its structure are known, but some model assumptions always need to be made. But it is well known that the proton structure can in fact have very dramatic effects on this impact factor. In the extreme case that the proton would exhibit a quark–diquark structure with a point-like diquark, for example, the impact factor even vanishes [13,63]. It was pointed out [63] that even a diquark cluster of a size as large as $0.3\,\mathrm{fm}$ could explain the experimental limit for the difference in the ratios of the real and imaginary part for pp and $p\bar{p}$ forward scattering. It is the aim of the present paper to study the influence of the proton structure on the odderon coupling and compare it with the available data for elastic pp and $p\bar{p}$ scattering in the dip region. As a framework we use the fit by Donnachie and Landshoff [10]. We replace the odderon–proton coupling used in that fit by a model for the proton structure which allows us to study the influence of a possible quark–diquark structure of the proton. The squared momentum transfer t in the dip region appears to be large enough to make the use of the simple picture of perturbative three gluon exchange possible, in which the odderon has intercept 1. A slow energy dependence of the odderon due to logarithmic enhancements should not have a sizable effect in the restricted range of energies for which data are available. In a similar way we also test other odderon–proton impact factors that have been used recently in diffractive η_c production. The crucial point is that we are now able to see whether they are compatible with the only existing data which clearly involve an odderon exchange.

This paper is organised as follows. In Sect. 2.1 we briefly sketch the original Donnachie–Landshoff fit and describe

in Sect. 2.2 two different models for the odderon–proton impact factor proposed in the literature. In Sect. 2.3 we present a geometrical model for the proton structure in position space. We show how this model can be implemented in a more general framework of high energy scattering in position space when applied to odderon exchange. In Sect. 3 we present the results for the differential cross section using these different models and confront them with the data. Finally, we give a brief summary and conclusions in Sect. 4.

2 Odderon–proton coupling and proton structure

In this section we discuss different ways in which the odderon–proton coupling can be described. We start by giving a short description of the Donnachie–Landshoff (DL) fit and its making use of the odderon. We then consider the perturbatively motivated description of the odderon–proton coupling via impact factors. These impact factors are usually used in perturbative calculations and most naturally written in momentum space. We discuss two different models of the impact factors that have been proposed in the literature. Finally we turn to a geometrical picture of the proton as a three-quark system in which we can easily study the effects of a possible quark–diquark structure in the proton. Obviously this geometrical model of the proton is most conveniently formulated in position space. We therefore find it useful to give a description of odderon exchange that works entirely in position space. We start from a general framework for high energy scattering and then derive a description of perturbative three gluon exchange in position space. The use of a simple picture of the odderon as an exchange of three perturbative gluons in the present paper is motivated by the fact that we are only considering the dip region of pp scattering at around $|t| \simeq 1.3\,\mathrm{GeV}^2$. This momentum transfer, however, is at the lower edge of the applicability of perturbation theory. A study of the dip region in a nonperturbative framework would therefore be very desirable. Although such a study is beyond the scope of the present paper we hope to pave the way for it by deriving the perturbative description of odderon exchange in position space in a more general framework which can also be used to implement nonperturbative models.

2.1 The Donnachie–Landshoff fit

A successful phenomenological description of all available pp and $p\bar{p}$ elastic scattering data in the framework of Regge theory was given by Donnachie and Landshoff [10]. This description is based on a number of exchanges in the t-channel: pomeron, reggeon, odderon, double pomeron, triple pomeron, pomeron plus two gluons, and reggeon plus pomeron. These exchanges are explicitly given as contributions to the scattering amplitude $T(s,t)$. For later use we would like to single out the odderon contribution $T^{\mathcal{O}}$ to that sum,
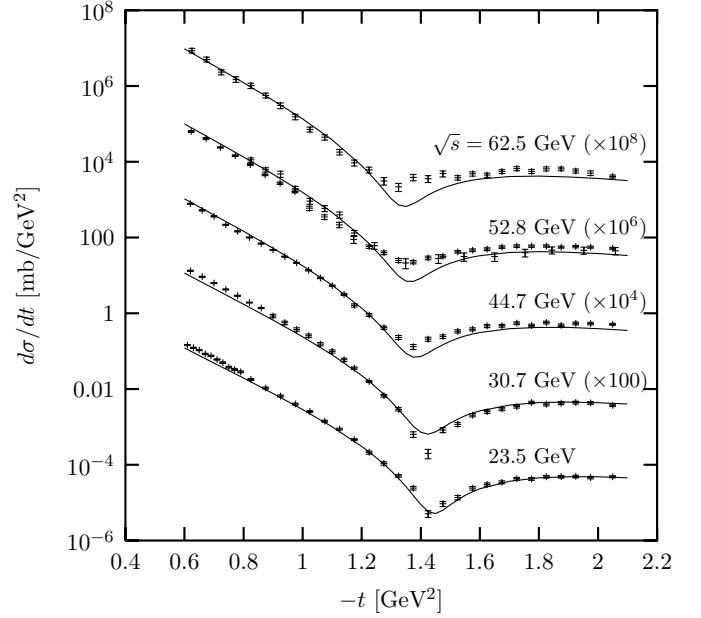

Fig. 2. The Donnachie–Landshoff fit for the differential elastic pp cross section

$$T(s,t) = T^{\mathcal{O}}(s,t) + T^{\mathrm{DL}}(s,t), \qquad (1)$$

where T^{DL} denotes all other contributions to the scattering amplitude, including the C-odd reggeon contribution. The differential cross section is then obtained from the scattering amplitude T via

$$\frac{\mathrm{d}\sigma}{\mathrm{d}t} = \frac{1}{16\pi s^2}\left|T(s,t)\right|^2. \qquad (2)$$

The different contributions to the scattering amplitude come with a number of parameters which have been fitted to all available data for elastic pp scattering in [10]. The details and all parameters can be found in that reference. In the present paper we do not attempt to improve the Donnachie–Landshoff fit. However, there appears to be a misprint in [10]. In order to reproduce a successful fit to the data the cutoff parameter t_1 for the gluon propagator as well as the parameter t_0 describing the charge distribution radius of the proton should be chosen as

$$t_0 = t_1 = 0.3\,\mathrm{GeV}^2, \qquad (3)$$

instead of $t_0 = t_1 = 300\,\mathrm{MeV}^2$ as given in (17) of the original paper [10]. With these changes in the original parameters we can reproduce the Donnachie–Landshoff fit. It is shown together with the relevant data in Fig. 2 where we have chosen to restrict ourselves to the dip region relevant for our study.

The odderon contribution is particularly important at large t and in the dip region. The dip originates from interference effects of the odderon contribution with other contributions, in particular with those of pomeron and double pomeron exchange. At large t the differential cross section is even dominated by odderon exchange, leading to the observed t^{-8} behaviour. In [10] the large-t data have
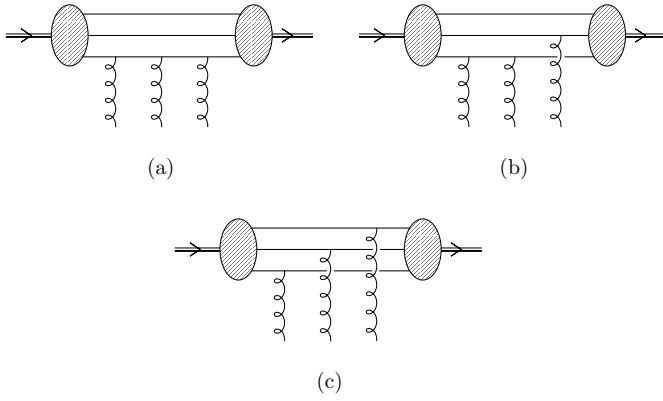
Fig. 3a–c. Diagrams contributing to the odderon–proton impact factor

therefore been used to fix the normalization parameter of the odderon contribution $T^{\mathcal{O}}$. In [64] it is argued that this dominance of the odderon at large t comes about because the exchange of three gluons permits one to distribute the momentum transfer evenly between the three quarks in the proton. Accordingly, that dominant contribution corresponds to a situation in which each of the three gluons is coupled to a different quark in the proton; see diagram (c) in Fig. 3. Also at smaller values of t the authors of [10] use a coupling of the odderon to the proton which is given only by this diagram. By selecting a single diagram only, however, gauge invariance is lost and the corresponding contribution becomes divergent as one of the gluon momenta goes to zero. Therefore the gluon propagator needs to be regularized by introducing the cut-off parameter t_1 (see above). With this procedure the coupling of the odderon to the proton used in [10] leads to a reasonably good agreement with the data also at intermediate values of t as shown in Fig. 2.

A note concerning the terminology used in [10] seems to be in order. In that paper the odderon contribution is called "three gluon exchange" but is in fact a pure $C = -1$ odderon exchange. In principle, it is of course possible to have three gluons in a $C = +1$ state, that is in an antisymmetric colour state. The fact that the three gluon exchange in the DL fit carries only $C = -1$ quantum numbers is due to the particular coupling of the three gluons to the proton chosen in [10], in which each of the three gluons is coupled to a different quark; see diagram (c) in Fig. 3. This immediately implies that the three gluons are in a symmetric colour state. (See also (28) and the corresponding discussion in Sect. 2.3 below.) However, gauge invariance requires one to include all possible ways of coupling the three gluons to the three quarks in the proton, in particular also diagrams of the type (a) and (b) in Fig. 3.

Finally, we would like to point out that the exchange of a three gluon state carrying positive C parity is not expected to change the DL fit. Due to reggeization a $C = +1$ perturbative state of three gluons has in the high energy limit the same analytic properties as a pomeron made of two gluons [65] (see also [47]). In a fit to the data such a contribution would therefore be absorbed by the

full pomeron exchange and is thus effectively already contained in the DL fit. It is therefore not necessary to consider a $C = +1$ three gluon exchange separately.

2.2 Impact factors

In the approach using impact factors the odderon contribution to the elastic proton–proton scattering amplitude is written in factorised form as

$$T^{\mathcal{O}}(s,t) = \frac{s}{32}\frac{5}{6}\frac{1}{3!}\int \frac{\mathrm{d}^2\delta_{1t}}{(2\pi)^4}\frac{\mathrm{d}^2\delta_{2t}}{(2\pi)^4}\left[\Phi_p(\vec{\delta}_{1t},\vec{\delta}_{2t},\vec{\Delta}_t)\right]^2$$
$$\times \frac{1}{\vec{\delta}_{1t}^2\,\vec{\delta}_{2t}^2(\vec{\Delta}_t-\vec{\delta}_{1t}-\vec{\delta}_{2t})^2}, \tag{4}$$

where the integral is over transverse momenta only. Here, $\vec{\Delta}_t$ is the total transverse momentum transferred in the t-channel, and $t = -\vec{\Delta}_t^2$. The last factor in the integral consists of the three gluon propagators which we assume to model the odderon at large t. The $5/6$ originates from a colour factor and the $1/3!$ is implied by the exchange of three identical gluons. The impact factor $\Phi_p(\vec{\delta}_{1t},\vec{\delta}_{2t},\vec{\delta}_{3t})$ describes the coupling of the odderon to the proton; it is not known from first principles. But some of its properties can be derived from general principles. In order to arrive at a gauge invariant expression for the impact factor one needs to take into account all possible ways to couple the three gluons to the three quarks in the proton. That means that one has include all three types of diagrams in Fig. 3 and the corresponding permutations of the gluon lines. The colour neutrality of the proton requires that the impact factor vanishes if one of the three transverse gluon momenta $\vec{\delta}_{it}$ vanishes,

$$\Phi_p(\vec{\delta}_{1t},\vec{\delta}_{2t},\vec{\delta}_{3t})\Big|_{\vec{\delta}_{it}=0} = 0, \quad i \in \{1,2,3\}, \tag{5}$$

which ensures that potential infrared singularities due to the gluon propagators in the integral (4) are cancelled. The above property implies that the impact factor has the general form

$$\Phi_p(\vec{\delta}_{1t},\vec{\delta}_{2t},\vec{\delta}_{3t})$$
$$= 8(2\pi)^2 g^3\left[F(\vec{\Delta}_t,0,0) - \sum_{i=1}^{3}F(\vec{\delta}_{it},\vec{\Delta}_t-\vec{\delta}_{it},0)\right.$$
$$\left. +2F(\vec{\delta}_{1t},\vec{\delta}_{2t},\vec{\delta}_{3t})\right], \tag{6}$$

where $\vec{\Delta}_t = \sum_{i=1}^{3}\vec{\delta}_{it}$, and $F(\vec{\delta}_{1t},\vec{\delta}_{2t},\vec{\delta}_{3t})$ is a form factor. The three terms in square brackets correspond (in the order given in (6)) to the diagram types (a), (b), and (c) in Fig. 3, respectively. The form factor F is related to the structure of the proton. Its exact form is unknown and needs to be modelled.

One model for the form factor F was given by Fukugita and Kwieciński in [66],

$$F(\vec{\delta}_{1t}, \vec{\delta}_{2t}, \vec{\delta}_{3t}) \tag{7}$$

$$= \frac{A^2}{A^2 + \frac{1}{2}\left[(\vec{\delta}_{1t} - \vec{\delta}_{2t})^2 + (\vec{\delta}_{2t} - \vec{\delta}_{3t})^2 + (\vec{\delta}_{3t} - \vec{\delta}_{1t})^2\right]}.$$

The parameter A is chosen to be half the ρ mass, $A = 384\,\text{MeV}$. This model for the form factor has recently been used in the estimate of the cross section for diffractive η_c photo- or electroproduction at HERA [48–50]. In these references a rather large value of $\alpha_s = g^2/(4\pi) = 1$ has been used for the strong coupling constant. This value was motivated by the use of a similar value in an estimate of hadronic cross sections in the two gluon model of [67].

Another model for the form factor F was proposed by Levin and Ryskin [14]. Their ansatz is motivated by a nonrelativistic quark model with oscillatory potential. Its explicit form is

$$F(\vec{\delta}_{1t}, \vec{\delta}_{2t}, \vec{\delta}_{3t}) = \exp\left(-R_p^2 \sum_{i=1}^3 \vec{\delta}_{it}^2\right). \tag{8}$$

The parameter R_p is supposed to be of the order of magnitude of the proton radius. In [14] a value of $2.75\,\text{GeV}^2$ is given for the quantity R_p^2. We assume that the misprint is located in the exponent of the units and thus use $R_p^2 = 2.75\,\text{GeV}^{-2}$. Assuming the missing minus sign to be in the exponent of R_p instead would imply an unusually small proton radius. The authors of [14] suggest to choose $\alpha_s = 1/3$.

2.3 High energy scattering in position space

In this subsection we give a very short recapitulation of the basic ideas of the treatment of high energy scattering developed by Nachtmann [68], for details and further justification we refer the reader to the original article. The method is based on the functional representation of scattering matrix elements and the WKB method. We first consider quark–quark scattering amplitudes in an external colour field using the WKB approximation. The quantisation is done by functional integration. Nucleon–nucleon scattering amplitudes are obtained from scattering amplitudes of clusters of quarks by averaging over wave functions in transverse space. This is an alternative to the treatment of high energy scattering in momentum space and particularly suited for investigating the effects of the spatial structure of the hadrons. In the present paper we use perturbative three gluon exchange to model the odderon, but the method presented here also allows one to easily incorporate nonperturbative models.

The first step is to transform the S-matrix element of two quarks with incoming momenta p_1, p_2 and outgoing momenta p_3, p_4 into a Green function. This is done by means of the LSZ reduction formalism,

$$\langle p_3 p_4^{\text{out}} | p_1 p_2^{\text{in}} \rangle = Z_\psi^{-2}$$
$$\times \int d^4x_1 \cdots d^4x_4 \exp\left[i(p_3 x_3 + p_4 x_4 - p_1 x_1 - p_2 x_2)\right]$$
$$\times \langle T\bar{u}(p_3)f(x_3)\bar{u}(p_4)f(x_4)\bar{f}(x_1)u(p_1)\bar{f}(x_2)u(p_2)\rangle, \tag{9}$$

where $f(x) = (i\gamma\partial - m)\psi(x)$ and Z_ψ is the wave function renormalization.

Next, the four point function $\langle T\psi(x_3)\psi(x_4)\bar{\psi}(x_1)\bar{\psi}(x_2)\rangle$ contained in the r.h.s. of (9) is expressed as a functional integral over the quark and the gluon fields, ψ and B respectively, written formally as

$$\langle T\psi(x_3)\psi(x_4)\bar{\psi}(x_1)\bar{\psi}(x_2)\rangle \tag{10}$$
$$= \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{D}B \psi(x_3)\psi(x_4)\bar{\psi}(x_1)\bar{\psi}(x_2) \exp[-iS_{\text{full QCD}}],$$

where $S_{\text{full QCD}}$ is the full QCD action. The fermion integration is Gaussian and can therefore be performed, yielding

$$\langle T\psi(x_3)\psi(x_4)\bar{\psi}(x_1)\bar{\psi}(x_2)\rangle = \int \mathcal{D}B \det[-i(i\gamma D - m)]$$
$$\times [S_F(x_3, x_1; B)S_F(x_4, x_2; B)$$
$$+ S_F(x_3, x_2; B)S_F(x_4, x_1; B)] \exp[-iS_{\text{pure QCD}}], \tag{11}$$

which contains the functional determinant of the Dirac operator, and the quark propagators $S_F(x_i, x_j; B)$ in the external colour potential B_μ^F. The functional integration is now to be performed only over the gluon fields with the pure QCD action (i.e. without quark contribution) in the measure. If we concentrate on lowest order exchange the determinant can be set to one. Furthermore, if we are interested only in momentum transfer small compared to the total energy the second (u-channel) term in the integrand can be neglected. Collecting all factors we finally obtain

$$\langle p_3 p_4^{\text{out}} | p_1 p_2^{\text{in}} \rangle \approx Z_\psi^{-2} \int \mathcal{D}B \mathcal{S}(p_3, p_1; B)\mathcal{S}(p_4, p_2; B)$$
$$\times \exp[-iS_{\text{pure QCD}}], \tag{12}$$

where $\mathcal{S}(p_i, p_j; B)$ is the scattering matrix element of a quark with momentum p_j to one with momentum p_i in an external colour field B.

In the next step we have to find a suitable form for the S-matrix element $\mathcal{S}(p_i, p_j; B)$. One can show [68] that the quark scattering matrix elements $\mathcal{S}(p_i, p_j; B)$ in an external field can be expressed as a generalized WKB expression

$$\mathcal{S}(p_i, p_j; B) = \bar{u}(p_i)\gamma^\mu u(p_j)P \exp\left[-ig\int_\Gamma \mathbf{B}_\rho dx^\rho\right]$$
$$\times \left(1 + O\left(\frac{1}{p_i^0}\right)\right), \tag{13}$$

where we denote by $\mathbf{B}$ the Lie-algebra valued gauge potential. The path-ordered integral is taken along the classical path Γ. From the scattering amplitudes for single quarks in the gluon field we obtain, according to (12), the nonperturbative quark–quark scattering amplitude by integrating the product of the two scattering amplitudes over the gluon field. More specifically, consider two quarks travelling along the light-like paths Γ_1 and Γ_2 given by

$$\Gamma_1 = (x^0, \vec{b}/2, x^3 = x^0) \quad \text{and}$$
$$\Gamma_2 = (x^0, -\vec{b}/2, x^3 = -x^0), \tag{14}$$

corresponding to quarks moving in opposite directions with the velocity of light, with an impact vector $\vec{b}$ in the $x^1 x^2$- plane (referred to in the following as the transverse plane). Let $\mathbf{V}_i(\pm\vec{b}/2)$ be the phases picked up by the quarks along these paths,

$$\mathbf{V}_i(\pm\vec{b}/2) = P \exp\left[-ig\int_{\Gamma_i}\mathbf{B}_\mu(z)\mathrm{d}z^\mu\right]. \quad (15)$$

Then the S-matrix element for two quarks with momenta p_1, p_2 and colour indices α_1, α_2 leading to two quarks of momenta p_3, p_4 and colours α_3, α_4 is [68]

$$S_{\alpha_3\alpha_4;\alpha_1\alpha_2}(s,t) = \bar{u}(p_3)\gamma^\mu u(p_1)\bar{u}(p_4)\gamma_\mu u(p_2)\mathcal{V}, \quad (16)$$

where

$$\mathcal{V} = iZ_\psi^{-2}\left\langle \int \mathrm{d}^2 b e^{-i\vec{q}\cdot\vec{b}}\left[\mathbf{V}_1\left(-\frac{\vec{b}}{2}\right)\right]_{\alpha_3\alpha_1}\right.$$
$$\left.\times\left[\mathbf{V}_2\left(+\frac{\vec{b}}{2}\right)\right]_{\alpha_4\alpha_2}\right\rangle. \quad (17)$$

Here $\langle\cdot\rangle$ denotes functional integration over the gluon field, and $\vec{q}$ is the momentum transfer $(p_1 - p_3)$ projected onto the transverse plane. Of course the approximation makes sense only if $|\vec{q}| \ll |\vec{p}|$.

In the limit of high energies we have helicity conservation,

$$\bar{u}(p_3)\gamma^\mu u(p_1)\bar{u}(p_4)\gamma_\mu u(p_2) \xrightarrow[s\to\infty]{} 2s\delta_{\lambda_3\lambda_1}\delta_{\lambda_4\lambda_2}, \quad (18)$$

where λ_i are the helicities of the quarks and $s = (p_1+p_2)^2$. In the following we can thus ignore the spin degrees of freedom.

In order to come to the nucleon–nucleon scattering amplitude we first consider the scattering of two groups of three quarks moving on parallel light-like world lines, each of which has the form

$$\hat{\Gamma}_1^a(x_0, \vec{b}/2 + \vec{x}_1^a, x^3 = x^0),$$
$$\hat{\Gamma}_2^a(x_0, -\vec{b}/2 + \vec{x}_2^a, x^3 = -x^0), \quad a = 1,2,3. \quad (19)$$

In order to ensure that these quark clusters asymptotically form colour singlet states all colours are parallel-transported in the remote past and future to a reference point of the cluster and there contracted antisymmetrically. This leads to the following S-matrix element [69] for scattering of colour-neutral clusters,

$$S\left(\vec{x}_1^1, \vec{x}_1^2, \vec{x}_1^3, \vec{x}_2^1, \vec{x}_2^2, \vec{x}_2^3\right) = \frac{1}{36}\frac{1}{Z_1 Z_2}$$
$$\times\left\langle \epsilon_{\alpha\beta\gamma}\left(\mathbf{V}_1^1\right)_{\alpha\alpha'}\left(\mathbf{V}_1^2\right)_{\beta\beta'}\left(\mathbf{V}_1^3\right)_{\gamma\gamma'}\epsilon_{\alpha'\beta'\gamma'}\epsilon_{\rho\mu\nu}\left(\mathbf{V}_2^1\right)_{\rho\rho'}\right.$$
$$\left.\times\left(\mathbf{V}_2^2\right)_{\mu\mu'}\left(\mathbf{V}_2^3\right)_{\nu\nu'}\epsilon_{\rho'\mu'\nu'}\right\rangle. \quad (20)$$

The non-Abelian phase factors $\mathbf{V}_i^a$ are defined as in (15) with the ⊔-shaped integration paths Γ_i as indicated in
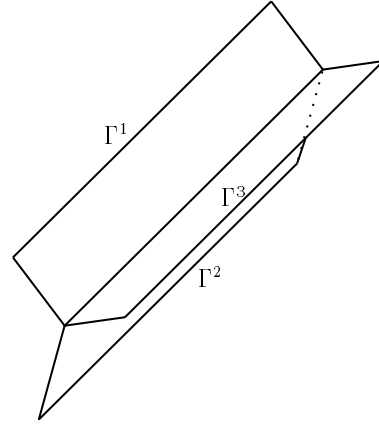

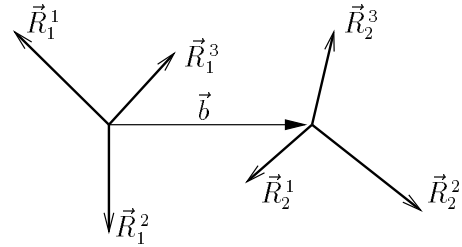**Fig. 4.** The paths in a colour-neutral three-quark cluster

Fig. 5. The positions of the quarks in transverse space

Fig. 4 for one cluster. The Z_i denote again the wave function renormalization for the respective clusters which in lowest order can be set equal to one. We introduce the reduced scattering amplitude J related to the S-matrix element for the scattering of quark clusters,

$$J(\vec{x}_1^1, \vec{x}_1^2, \vec{x}_1^3, \vec{x}_2^1, \vec{x}_2^2, \vec{x}_2^3) = S(\vec{x}_1^1, \vec{x}_1^2, \vec{x}_1^3, \vec{x}_2^1, \vec{x}_2^2, \vec{x}_2^3) - 1. \quad (21)$$

The differential nucleon–nucleon cross section is obtained from the gauge invariant scattering amplitude $T(s, t)$ via (2). The odderon contribution $T^\mathcal{O}(s,t)$ to the scattering amplitude $T(s,t)$ is computed by integrating over the transverse coordinates with a suitable transverse wave function ψ,

$$T^\mathcal{O}(s,t) = 2is\int \mathrm{d}^2 b e^{-i\vec{q}\cdot\vec{b}}\int \mathrm{d}^6\mathbf{R}_1\mathrm{d}^6\mathbf{R}_2|\psi(\mathbf{R}_1)|^2|\psi(\mathbf{R}_2)|^2$$
$$\times J(\vec{x}_1^1, \vec{x}_1^2, \vec{x}_1^3, \vec{x}_2^1, \vec{x}_2^2, \vec{x}_2^3), \quad (22)$$

where the colour indices in (21) are symmetric, and $\mathbf{R}_i$ denotes the set of positions of the quarks relative to the centre of nucleon i, and $\vec{b}$ is the impact vector between the two nucleons (see Fig. 5),

$$\mathbf{R}_i = \left(\vec{R}_i^1, \vec{R}_i^2, \vec{R}_i^3\right), \quad \vec{x}_1^a = \frac{\vec{b}}{2} + \vec{R}_1^a, \quad \vec{x}_2^a = -\frac{\vec{b}}{2} + \vec{R}_2^a. \quad (23)$$

For a perturbative evaluation of three gluon exchange we expand $\mathbf{V}_i^a$ in (20) up to order g^3. Expanding the matrix valued path integral in generators τ of $su(3)$,

$$\int_{\Gamma_i^a}\mathrm{d}z^\mu\mathbf{B}_\mu(z) = \hat{B}_{a,i}^c\tau^c, \quad (24)$$

we obtain

$$
(\mathbf{V}_i^a)_{\alpha\beta} = \delta_{\alpha\beta} - \mathrm{i}g\hat{B}_{a,i}^c \tau_{\alpha\beta}^c - \frac{1}{2}g^2 \hat{B}_{a,i}^c \hat{B}_{a,i}^{c'} (\tau^c \tau^{c'})_{\alpha\beta}
$$
$$
- \frac{\mathrm{i}^3}{3!}g^3 \hat{B}_{a,i}^c \hat{B}_{a,i}^{c'} \hat{B}_{a,i}^{c''} (\tau^c \tau^{c'} \tau^{c''})_{\alpha\beta} + \mathcal{O}(g^4). \quad (25)
$$

In principle we have to take into account path ordering in the integral (24). But since we are only interested in the odderon contribution, which is symmetric in the colour indices, we may discard it here. To that order we also do not need to take into account the parallel transporters from the reference points to the light-like paths and we have set $Z_i = 1$.

The lowest order three gluon exchange contribution to (20) is obtained by pairing three fields of group (1) with three of group (2). In each group we have three possibilities:

(a) two quarks are not involved;
(b) one quark is not involved;
(c) all quarks are involved.

We consider first the respective colour tensors C^a, C^b, C^c. For case (a) we have

$$
C^\mathrm{a}_{cc'c''} = \epsilon_{\alpha\beta\gamma}\epsilon_{\alpha'\beta'\gamma'}\delta_{\alpha\alpha'}\delta_{\beta\beta'}(\tau^c \tau^{c'} \tau^{c''})_{\gamma\gamma'}
$$
$$
= \frac{1}{2}d_{cc'c''} + \frac{\mathrm{i}}{2}f_{cc'c''}, \quad (26)
$$

where f are the structure constants of $su(3)$ and d the symmetric constants occurring in the anti-commutators. For case (b) we obtain in the same way the colour factor

$$
C^\mathrm{b}_{cc'c''} = -\frac{1}{4}d_{cc'c''} - \frac{\mathrm{i}}{4}f_{cc'c''}, \quad (27)
$$

and for the case where all quarks in the nucleon occur in pairings

$$
C^\mathrm{c}_{cc'c''} = \frac{1}{2}d_{cc'c''}. \quad (28)
$$

Only the symmetrically coupled colours contribute to $C = -1$ exchange. Since in the treatment of DL only case (c) was considered they automatically had only a negative charge parity contribution (see above). We are only interested in the $C = -1$ contribution and because of the symmetry the path ordering has no influence. This simplifies the calculation considerably, since now we can perform the integrations along the light-like paths without restriction and this leads to a projection into the transverse space. The general structure of a contribution to (20) is therefore given by the product of two colour factors C given above and the product of three gluon propagators in transverse space connecting a quark of group (1) with one of group (2).

We therefore obtain for the reduced scattering amplitude

$$
J(\vec{x}_1^1, \vec{x}_1^2, \vec{x}_1^3; \vec{x}_2^1, \vec{x}_2^2, \vec{x}_2^3)
$$
$$
= g^6 \sum_{a_i,b_i=1}^{3} K(a_1, a_2, a_3; b_1, b_2, b_3)
$$
$$
\times \; \chi(\vec{x}_1^{a_1}, \vec{x}_2^{b_1})\chi(\vec{x}_1^{a_2}, \vec{x}_2^{b_2})\chi(\vec{x}_1^{a_3}, \vec{x}_2^{b_3}). \quad (29)
$$

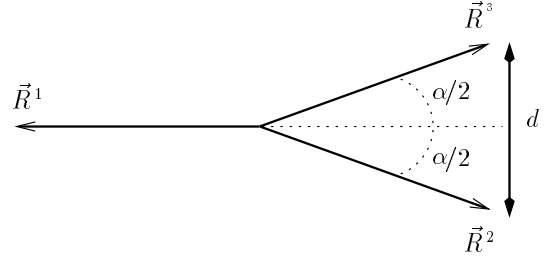**Fig. 6.** Definition of the angle α characterising the proton configuration

The factor K is obtained from (20), (21) and (25) taking into account only colour symmetric gluon states. The corresponding contractions and the combinatorial factors arising in the sum over indices in (29) have been calculated in [63,70]. We refer the reader to [70] for the somewhat cumbersome details. χ is the gluon propagator in transverse space,

$$
\chi(\vec{x}, \vec{y}) = \int \frac{\mathrm{d}^2 k}{(2\pi)^2} \frac{1}{\vec{k}^2 + m^2} \mathrm{e}^{-\mathrm{i}\vec{k}\cdot(\vec{x}-\vec{y})} \quad (30)
$$
$$
= \frac{1}{2\pi} K_0\left(m\,|\vec{x} - \vec{y}|\right), \quad (31)
$$

where K_0 is the modified Bessel function. The single diagrams are infrared divergent. In order to regularise them we have introduced a gluon mass m, which is possible in LO approximation. In the final gauge invariant expressions we can set the gluon mass to zero.

For the quark density in the nucleon we make the simple ansatz

$$
\left|\psi(\vec{R}_1, \vec{R}_2, \vec{R}_3)\right|^2 \quad (32)
$$
$$
= \frac{2}{\pi}\frac{1}{S_p^2} \exp\left(-\frac{2R_1^2}{S_p^2}\right) \delta^2(\vec{R}_2 - \mathbf{M}_\beta \vec{R}_1)\delta^2(\vec{R}_3 - \mathbf{M}_{-\beta}\vec{R}_1),
$$

where

$$
\mathbf{M}_\beta = \begin{pmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{pmatrix}
$$

and $\beta = \pi - \alpha/2$. The quantity S_p determines the electromagnetic radius of the nucleon. We choose $S_p = 0.8\,\mathrm{fm}$ in the range given in [70,71]. The meaning of the angle α is illustrated in Fig. 6. The value $\alpha = 2\pi/3$ corresponds to a Mercedes star configuration of the quarks in the nucleon. $\alpha = 0$ corresponds to a quark–diquark picture of the nucleon with an exactly point-like diquark. For small angles α we can still speak of a diquark–cluster in the nucleon, and we call the distance d between the two quarks in such a cluster the diquark size; see Fig. 6. With the wave function (32) one then obtains for the average diquark size $\langle d \rangle$

$$
\langle d \rangle = \sqrt{\frac{\pi}{2}} \sin\left(\frac{\alpha}{2}\right) S_p. \quad (33)
$$

3 Results

In the following we use the Donnachie–Landshoff fit as a framework for confronting different models for the cou-
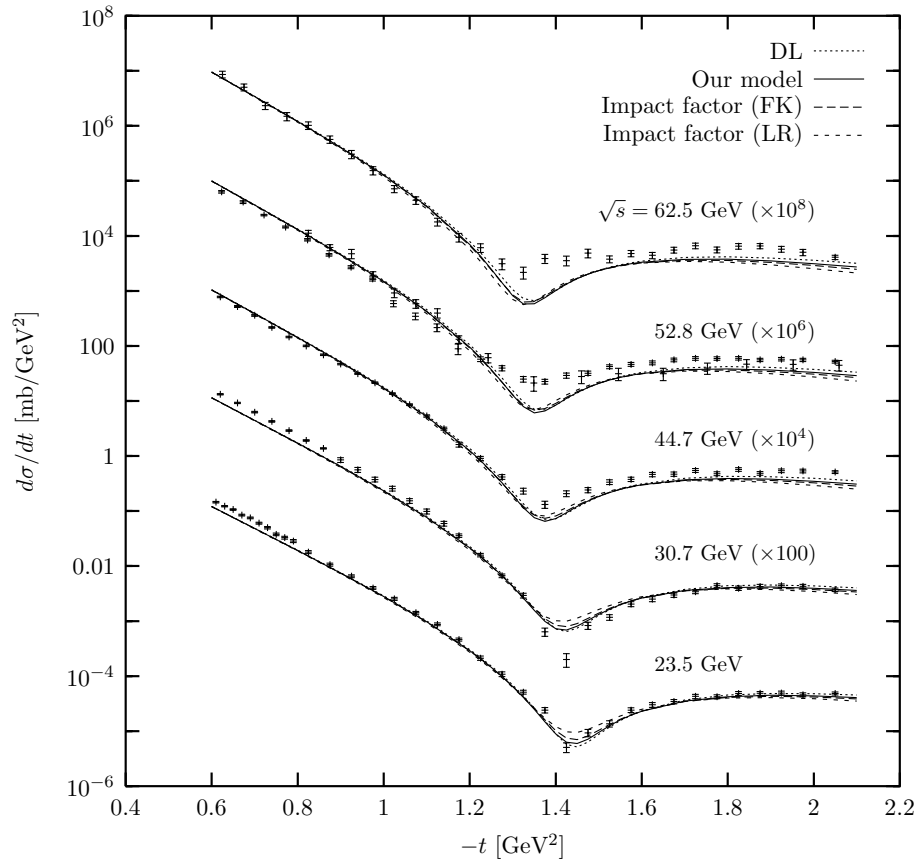
Fig. 7. Differential cross section for elastic pp scattering calculated using different couplings of the odderon to the proton: the original Donnachie–Landshoff fit (*dotted*), our geometrical model for the proton (*solid*), and the Fukugita–Kwieciński (FK, *long-dashed*) and Levin–Ryskin (LR, *short-dashed*) impact factors

pling of the odderon to the proton with the pp and $p\bar{p}$ elastic scattering data in the dip region. This is done by replacing the odderon contribution $T^{\mathcal{O}}$ to the scattering amplitude T in the DL fit by the other models for $T^{\mathcal{O}}$ discussed in Sects. 2.2 and 2.3. The integrations in the calculation of the differential cross section are performed numerically.

The results for the differential cross section in the dip region are shown in Fig. 7 together with the relevant data. For comparison we also show the Donnachie–Landshoff fit described already in Sect. 2.1 as the dotted line in this figure.

The solid line in Fig. 7 represents the result obtained with the geometric model for the proton described in Sect. 2.3. It almost coincides with the DL fit and gives a satisfactory description of all available data. We have fixed the value of the strong coupling at $\alpha_s = 0.4$ and then adjusted the angle α characterising the proton configuration. The optimal description of the data is obtained for $\alpha = 0.14\pi$, corresponding to an average diquark size of $0.22\,\text{fm}$. For other choices of the average diquark size (or equivalently of the angle α) and fixed $\alpha_s = 0.4$ the description of the data becomes much worse as is illustrated in Fig. 8 for one centre-of-mass energy, $s^{1/2} = 44.7\,\text{GeV}$. The situation is very similar for the other centre-of-mass energies. With increasing average diquark size $\langle d \rangle$ the minimum of the differential cross section moves towards smaller t. At the same time the depth of the dip changes in such a way
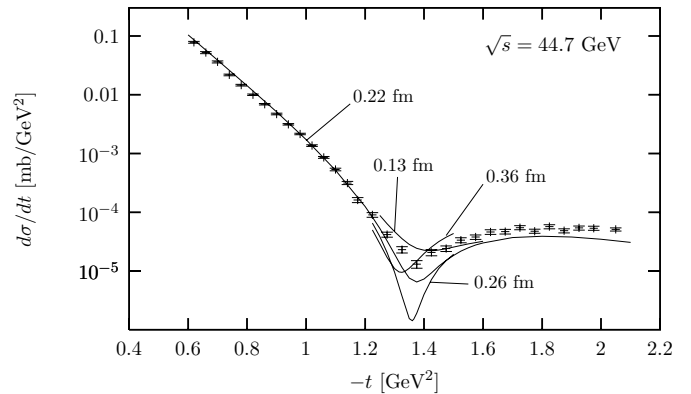

Fig. 8. Dependence of the differential cross section on the average diquark size $\langle d \rangle$ chosen in our geometric model of the proton for fixed coupling constant $\alpha_s = 0.4$

that the optimal value of $\langle d \rangle$ can be determined with only a small uncertainty.

The parameters α_s, α, and S_p are of course strongly correlated in their effect on the differential cross section. Since S_p is rather strictly constrained by the electromagnetic size of the nucleon we do not vary it here. The constraints on the other two parameters in our model, on the other hand, are only weak. The correct value of the strong coupling constant α_s is not known precisely in the dip region but has a strong effect on the cross section as it enters
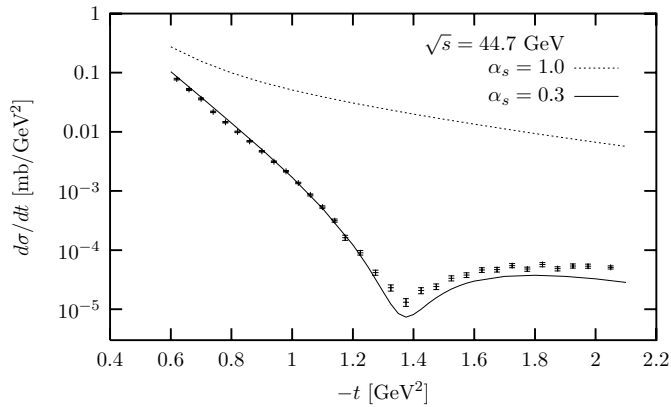
Fig. 9. Dependence of the differential cross section obtained from the Fukugita–Kwieciński impact factor on the choice of α_s

Fig. 10. Differential cross section for elastic $p\bar{p}$ scattering at $s^{1/2} = 53$ GeV as calculated using different couplings of the odderon to the proton: the original Donnachie–Landshoff fit (*dotted*), our geometrical model for the proton (*solid*), and the Fukugita–Kwieciński (FK, *long-dashed*) and Levin–Ryskin (LR, *short-dashed*) impact factors; data taken from [5]

in the third power on the amplitude level already. The correct value for the angle α is even less constrained, and also the variation of α has a strong effect on the cross section. This is particularly true for small values of α (or small diquark sizes), which are known to imply a strong suppression of the amplitude. In the framework of our present investigation it is obviously not possible to determine α_s and the angle α independently. We have therefore determined the optimal value for α also for other choices of α_s than the one mentioned above. For the choice $\alpha_s = 0.3$, for instance, we find that the best description of the data results for $\alpha = 0.22\pi$, corresponding to an average diquark size of $\langle d \rangle = 0.34$ fm. Choosing $\alpha_s = 0.5$ instead, we find an optimal value of $\alpha = 0.095\pi$, corresponding to $\langle d \rangle = 0.15$ fm. We would like to point out that the resulting sizes of the diquark cluster in the nucleon are thus rather small for all reasonable choices of α_s at the relevant momentum scale in the dip region. A Mercedes star configuration in the proton would in fact imply an unrealistically small value of $\alpha_s \simeq 0.17$. This result of course assumes that LO perturbation theory can be applied in the dip region.

We now turn to the models for the odderon–proton impact factor described in Sect. 2.2. Both models contain two parameters one of which is the strong coupling α_s. The other one is in the case of the Fukugita–Kwieciński (FK) model the parameter $A = m_\rho/2$, in the case of the Levin–Ryskin (LR) model it is the parameter R_p. The latter parameters are again related to the proton size and should thus be considered strongly constrained. We therefore keep them at the values given in the original papers (as quoted in Sect. 2.2) and vary only α_s. The differential cross section obtained with the Fukugita–Kwieciński model (7) for the impact factor is shown as the long-dashed curve in Fig. 7. It gives an equally good description of the data as the DL fit and as our geometric model of the proton. In order to obtain this curve we have chosen $\alpha_s = 0.3$ instead of the value $\alpha_s = 1.0$ originally proposed in [66]. Had we chosen that value instead, the resulting differential cross section would dramatically overshoot the data and not even show a dip structure, as is illustrated
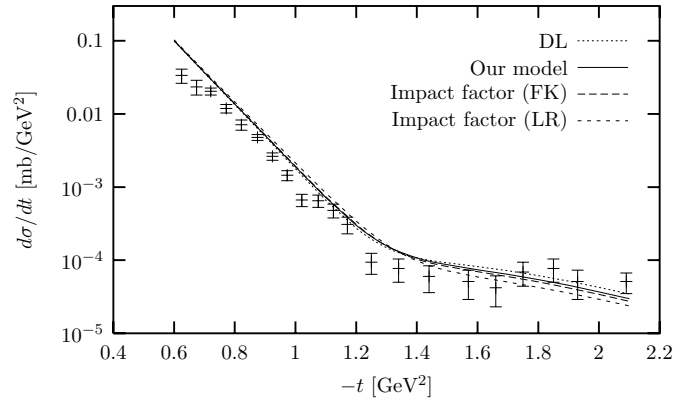
for one centre-of-mass energy ($s^{1/2} = 44.7$ GeV) in Fig. 9. Also the Levin–Ryskin model (8) for the impact factor leads to a good description of the data when the strong coupling constant is chosen as $\alpha_s = 0.5$. The corresponding differential cross section is shown as the short-dashed curve in Fig. 7. Also in this case the dependence of the cross section on α_s is very strong, actually being the same as in the case of the FK model as can easily be seen from (6).

Finally, we turn to the differential cross section for elastic $p\bar{p}$ scattering. Unfortunately, sufficiently many data points are available only for one centre-of-mass energy in the ISR range, $s^{1/2} = 53$ GeV. Our results for that energy are shown in Fig. 10. Here we have used the same parameters as for the curves in Fig. 7. Again, our geometric model as well as the two models for the impact factors lead to a description of the data which is as good as the Donnachie–Landshoff fit, producing a shoulder rather than the dip observed in pp scattering.

In summary we can say that the experimental data available in the dip region are by far not precise enough to distinguish between different models for the coupling of the odderon to the proton. All models for that coupling and the corresponding models for the proton structure lead to a satisfactory description of the data when the respective parameters are chosen appropriately. But for a given model these parameters are quite strongly constrained by the data. This applies in particular to the value of α_s in the two models using impact factors.

4 Conclusions

The only clear experimental evidence for the existence of an odderon comes from measurements of the differential cross section for high energy elastic pp and $p\bar{p}$ scattering in the dip region at around $|t| \simeq 1.3$ GeV^2. The odderon contribution to this process is expected to be sensitive to

the proton structure. In the present paper we have studied different models for the odderon–proton coupling. As a framework we have used the Donnachie–Landshoff fit which successfully describes all available data for this process, and we have replaced the odderon contribution to this fit by the respective model. We have taken two models for the odderon–proton coupling from the literature. These two models are based on impact factors in momentum space. In addition, we have constructed a geometric model for the proton in which the effect of a possible diquark cluster can be studied. In all three cases the odderon is modelled by perturbative three gluon exchange in the $C = -1$ channel.

We find that all models for the odderon–proton coupling give very similar results if the model parameters, in particular the strong coupling constant, are chosen appropriately. All models work as well as the original Donnachie–Landshoff fit. The available data cannot distinguish between the different models. But for a given model the data impose very strong constraints on the parameters of that model. Using our geometric model we find that the average size of the diquark cluster in the proton is quite small, $\langle d \rangle < 0.35$ fm. This result is obtained when assuming that reasonable values for strong coupling constant α_s in the dip region are larger than 0.3. In the nonperturbative model used in [63] such a small diquark is sufficient to explain the absence of an odderon signal in the ratio of the real to imaginary part in the forward direction [4]. This can be understood easily. In the nonperturbative model for the IR behaviour of QCD soft gluons dominate and therefore the resolution is much coarser.

It turns out that in the models based on odderon–proton impact factors the data impose rather strong constraints on the choice of the strong coupling constant α_s which appears as a parameter in these models. In the case of the impact factor proposed by Levin and Ryskin we find that α_s has to be chosen as 0.5, i.e. a value rather close to the 1/3 proposed originally.

Of particular interest is the model for the odderon–proton impact factor proposed by Fukugita and Kwieciński. Recently, this model has been used for the calculation of different processes, among them the diffractive photo- and electroproduction of η_c mesons at HERA. This process is currently considered to be one of the best possible ways to observe the odderon experimentally. The corresponding calculations [48–50] use a rather large value $\alpha_s = 1$ in the impact factor. In order to describe the data for *pp* elastic scattering with this impact factor, however, we find that α_s needs to be chosen as 0.3. This observation indicates that the current estimates for diffractive η_c production at HERA might be somewhat optimistic.

In our study we have assumed that the odderon can be modelled by perturbative three gluon exchange. However, the dip region of *pp* elastic scattering is located at momentum transfers $t^{1/2}$ just slightly above 1 GeV, that is at the lowest edge of the applicability of perturbation theory. It would therefore be very desirable to study this process also in the framework of a nonperturbative model.

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